

**Math-M-Addicts Group S-L Entrance Exam (SAMPLE)**

**Instructions:** The Group S exam consists of questions 1-6; the Group L exam consists of questions 4-9. If you know which exam you would like to take, solve only the questions for that exam. If you would like to test for both exams, attempt all of the questions! Admission to each Group will be determined separately, and will be based solely on the questions for that Group.

MMA Only:

S: \_\_\_\_\_

L: \_\_\_\_\_

Please write legibly and fully justify your answers to receive partial credit.

**S** **Problem 1:** Numbers 1, 2, 3, ..., 20 were written on a whiteboard. Erika erased two of the numbers, so that the remaining eighteen numbers add up to 183. What is the largest possible value of the difference between the two erased numbers?

**S** **Problem 2:** A box contains four different pairs of socks: one red pair, one blue pair, one purple pair and one green pair. Joe took 4 random socks out of the box. What is the probability that all the socks that Joe took are of different colors (i.e. one red, one blue, one purple and one green)?

**S** **Problem 3:** A positive integer  $N$  is a multiple of 51. When  $\frac{N}{3}$  is divided by  $\frac{N}{17}$  with remainder, that remainder is equal to 20 (note that both  $\frac{N}{3}$  and  $\frac{N}{17}$  are integers). Show that there is a unique natural number  $N$  satisfying these conditions and find it.

**S/L** **Problem 4:** There are a total of 47 students in Math-m-addicts S groups. Is it possible that each of them has exactly 1, 5, or 9 friends amongst those 47? (In this problem, we treat friendships as symmetric; i.e. if Bob is Peter's friend then Peter is also Bob's friend).

**S/L** **Problem 5:** In a number  $N = 17?552?8$  the two question marks stand for unknown digits that do not have to be the same. It is known that  $N$  is a multiple of 72. Find all possible values of  $N$ .

**S/L** **Problem 6:** Polling company Opinions & Co talked to 50 residents of a town that held an election of a mayor. Each resident voted for precisely one candidate. Prove that Opinions & Co can find either 8 residents who voted for the same candidate or 8 residents who voted for 8 distinct candidates.

**L** **Problem 7:** A teacher wrote out the first few odd numbers 1, 3, 5, ... on a whiteboard. Mary erased one of the numbers. The remaining numbers add up to 700. Which number was erased by Mary? Find all possibilities and prove that there are no others.

**L** **Problem 8:** Department of Transportation has 9 parking spots, arranged in a straight line. A parked truck occupies three adjacent spots, a parked bus occupies two adjacent spots while a parked car occupies one spot. How many ways are there to park a green truck, a yellow bus, a red bus and a blue car into those 9 spots?

**L** **Problem 9:** 5 rooks are placed on a 5x5 chessboard in such a way that no two rooks attack each other. Prove that there exists a 2x2 square on the chessboard which does not contain any of the rooks.