## Math-M-Addicts May 2018 Group S Entrance Exam

**Instructions**: Please write legibly and fully justify your answers. Points will be deducted for incomplete solutions. At the same time, it is OK to provide partial answers, as you may earn points for good ideas even if you do not have full solutions. Good luck!

**Problem 1**. Two prime numbers were written on a whiteboard. Each of these numbers was increased by one. This caused their product to increase by exactly 100. Which numbers were originally written on the board?

**Problem 2.** There are six trails leading to the top of Mount Math. Sinai makes two hikes up and down the mountain, one in the morning and one in the afternoon. On each hike, she uses a different trail to descend than the trail she used to ascend, though the same trail can be used in both the morning and the afternoon hikes. In how many ways can Sinai plan her hikes?

**Problem 3**. Each cell of a 3x3 square contains a number. The sum of numbers in each row equals 6. The sum of numbers in each column also equals 6. The sum of numbers in any 2x2 square equals 7. Determine, with proof, the number written in the central square.

**Problem 4.** Place numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 around a circle so that sum of any two numbers next to each other is not divisible by 3, 5 or 7.

**Problem 5.** How many two-digit numbers give a perfect square when added to its "mirror" (the two-digit number written with its digits in reverse order)?

**Problem 6.** A carpenter went to the store and bought 10 planks of wood. Each plank has a length that is a whole number of centimeters. The longest plank has a length of exactly 54 centimeters. Prove that there exist three planks that can be arranged to form a triangle.